

The chiral magnetic effect in hydrodynamical approach

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Abstract

We consider a model with two conserved currents, vector and axial, and two associated chemical potentials, μ and μ_5 . In the presence of external magnetic and electric fields the axial current is anomalous. Generalizing recent results and using thermodynamic relations alone we demonstrate that one can evaluate the chiral magnetic effect (ChME), that is the electric current flowing in the direction of the external magnetic field. In the linear in the chemical potential approximation the current is the same as for non-interacting fermions. In other words, there exists a hydrodynamic non-renormalization theorem for the ChME.

1 Introduction

The chiral magnetic effect (for review and references see [1]) gradually becomes a litmus-paper for quark-gluon plasma physics. So much the more interesting is the task of its explanation and study. The effect itself is the generation of electric current by applying an external magnetic field \mathbf{H} to a chiral medium:

$$\mathbf{j}_{el} = c_{ch} \mathbf{H} , \quad (1)$$

where c_{ch} is a constant. The problem is to evaluate the coefficient c_{ch} . In particular, for fermions interacting with the magnetic field but not among themselves one gets [1],[2]:

$$c_{ch} = N_c \frac{e^2 \mu_5}{2\pi^2} , \quad (2)$$

where N_c is the number of colors, e^2 is the electric charge squared and μ_5 is the chemical potential conjugated to the the difference in the number of left- and right-handed fermions. Taking into account the interaction between the fermions is a challenge.

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In our considerations, we start from the hydrodynamic approximation which is, in a way, opposite to the limit of free fermions. Indeed, in the hydrodynamic approximation one considers distances l much larger than the free path of the particles of the medium,

$$l \gg l_{free \ path} \ .$$

The hydrodynamic approach to chiral media is pioneered in [3] where it is demonstrated that the general thermodynamic identities are powerful enough to fix some transport coefficients. In the model of [3] all the currents are chiral. We generalize these considerations to the case of vector and axial currents separately conserved (at least in the absence of external electromagnetic fields). The two-current set-up (see also [4]) enables physical quantities to have their proper parity.

With all the calculations completed, we obtain the first term in the expansion of the coefficient c_{ch} in the axial chemical potential μ_5 , whereas in coupling constant it includes all orders due to our use of the hydrodynamical approximation. The result coincides with (2). The fact that calculated coefficient coincides with those derived in linear approximation permits us to formulate a non-renormalization theorem for ChME that seems to be analogous to already existing non-renormalization theorems [5]: the ChME coefficient doesn't receive corrections in coupling constant, but only in chemical potentials of right- and left-handed particles. In fact, this theorem is a consequence of the second law of thermodynamics.

Through the paper we use the following conventions: the metric tensor has $(-,+,+,+)$ signature, 4-velocity of an element of the liquid satisfies $u_\mu u^\mu = -1$, the quantities which correspond to the electric and magnetic fields in the rest frame are $E^\mu = F^{\mu\nu} u_\nu$ and $B_\mu = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} u^\nu F^{\alpha\beta}$. We also introduce the following notation:

$$\omega_\mu = \frac{1}{2} \epsilon_{\mu\nu\alpha\beta} u^\nu \partial^\alpha u^\beta \ .$$

For simplicity we consider an Abelian case henceforth ($N_c = 1$).

2 Hydrodynamical approximation

We discuss a hydrodynamical model of the ideal chiral liquid in the presence of the chiral anomaly. To describe a nonzero chirality we introduce independent densities of the right- and left-handed particles or, equivalently, density n and "axial" density n_5 (which phenomenologically correspond to the QCD currents $J^\mu = \bar{q}\gamma^\mu q$ and $J_5^\mu = \bar{q}\gamma^\mu \gamma^5 q$). We also introduce their chemical potentials μ, μ_5 . This model is described by following equations:

$$\begin{aligned} \partial_\mu T^{\mu\nu} &= F^{\nu\lambda} j_\lambda \\ \partial_\mu j^\mu &= 0 \\ \partial_\mu j_5^\mu &= C E^\mu B_\mu, \end{aligned} \tag{3}$$

where the stress-energy tensor, vector and axial current are

$$\begin{aligned} T^{\mu\nu} &= wu^\mu u^\nu + pg^{\mu\nu} \\ j^\mu &= nu^\mu + \nu^\mu \\ j_5^\mu &= n_5 u^\mu + \nu_5^\mu \end{aligned} \quad . \quad (4)$$

Terms ν, ν_5 are higher-order corrections in velocities and their derivatives, which corresponds, in particular, to dissipative effects. There is no a dissipative force in the rest frame of the element of a liquid. Therefore ν, ν_5 satisfy conditions(see [6]):

$$\begin{aligned} \nu_\mu u^\mu &= 0 \\ \nu_{5,\mu} u^\mu &= 0. \end{aligned} \quad (5)$$

Following the pattern of [3], let us consider consequences from the second law of thermodynamics. Transforming $u_\nu \partial_\mu T^{\mu\nu} + \partial_\mu j^\mu + \partial_\mu j_5^\mu$ we get:

$$\partial_\mu (su^\mu - \frac{\mu}{T}\nu^\mu - \frac{\mu_5}{T}\nu_5^\mu) = -\nu^\mu \partial_\mu \frac{\mu}{T} - \nu_5^\mu \partial_\mu \frac{\mu_5}{T} + \frac{E^\lambda}{T}\nu_\lambda - \mu_5 \frac{C}{T} E \cdot B. \quad (6)$$

The second law of thermodynamics implies that the entropy is non-decreasing and the entropy current divergence is non-negative. The entropy current is usually introduced as:

$$s^\mu = su^\mu - \frac{\mu}{T}\nu^\mu - \frac{\mu_5}{T}\nu_5^\mu. \quad (7)$$

For the ideal liquid ν and ν_5 are zero. But in the presence of the anomaly such a choice does not guarantee fulfilment of the second law of thermodynamics since the $CE \cdot B$ term can have both signs. To compensate for the contribution of this term we can introduce in ν, ν_5 and in s_μ further terms, not forbidden by the second law of thermodynamics in the presence of the anomaly. We choose such terms in the accordance with [3]:

$$s^\mu = su^\mu - \frac{\mu}{T}\nu^\mu - \frac{\mu_5}{T}\nu_5^\mu + D\omega^\mu + D_B B^\mu, \quad (8)$$

$$\begin{aligned} \nu^\mu &= \kappa\omega^\mu + \kappa_B B^\mu \\ \nu_5^\mu &= \xi\omega^\mu + \xi_B B^\mu. \end{aligned} \quad (9)$$

Coefficients $\xi, \xi_B, \kappa, \kappa_B, D, D_B$ are functions of the thermodynamical variables. The coefficient in front of B^μ in the vector current corresponds to the ChME in hydrodynamical approach. To impose the second law of thermodynamics we demand the contributions to the divergence of the new terms to be compensated by the chiral anomaly. When B^μ, ω^μ, E, B are arbitrary, the following set of equations takes place:

$$(\partial_\mu D - 2\frac{\partial_\mu P}{w}D - \kappa\partial_\mu\frac{\mu}{T} - \xi\partial_\mu\frac{\mu_5}{T}) \cdot \omega^\mu = 0 \quad (10)$$

$$(\partial_\mu D_B - \frac{\partial_\mu P}{w}D_B - \kappa_B\partial_\mu\frac{\mu}{T} - \xi_B\partial_\mu\frac{\mu_5}{T}) \cdot B^\mu = 0 \quad (11)$$

$$(\frac{2nD}{w} - 2D_B + \frac{\kappa}{T}) \cdot (E_\mu\omega^\mu) = 0 \quad (12)$$

$$(\frac{nD_B}{w} + \frac{\kappa_B}{T} - \mu_5\frac{C}{T}) \cdot (E_\mu B^\mu) = 0. \quad (13)$$

To solve this set of equations one should use initial conditions. For this purpose it is convenient to change a thermodynamical variables from (μ, μ_5, T) to $(\bar{\mu} = \frac{\mu}{T}, \bar{\mu}_5 = \frac{\mu_5}{T}, P)$. Then

$$dT = \frac{T}{w}dP - \frac{nT^2}{w}d\bar{\mu} - \frac{n_5T^2}{w}d\bar{\mu}_5. \quad (14)$$

We can set the gradients of thermodynamical variables $(\partial P, \partial_T^\mu, \partial_T^{\mu_5})$ arbitrary at initial time. Then coefficients in front of them must be zero at arbitrary time to make (10-13) valid :

$$\frac{\partial D}{\partial p} - \frac{2D}{w} = 0, \quad \frac{\partial D_B}{\partial p} - \frac{D_B}{w} = 0 \quad (15)$$

$$\frac{\partial D}{\partial \bar{\mu}} = \kappa, \quad \frac{\partial D_B}{\partial \bar{\mu}} = \kappa_B \quad (16)$$

$$\frac{\partial D}{\partial \bar{\mu}_5} = \xi, \quad \frac{\partial D_B}{\partial \bar{\mu}_5} = \xi_B. \quad (17)$$

We search for a solution in the form:

$$\begin{aligned} D &= T^2 d(\bar{\mu}, \bar{\mu}_5), \quad D_B = T d(\bar{\mu}, \bar{\mu}_5) \\ \kappa &= \frac{\partial}{\partial \bar{\mu}}(T^2 d), \quad \xi = \frac{\partial}{\partial \bar{\mu}_5}(T^2 d) \\ \kappa_B &= \frac{\partial}{\partial \bar{\mu}}(T d_B), \quad \xi_B = \frac{\partial}{\partial \bar{\mu}_5}(T d_B), \end{aligned} \quad (18)$$

where d and d_B are arbitrary functions. One finds after straightforward calculations:

$$d_B = C\bar{\mu}_5\bar{\mu} + f(\bar{\mu}_5) \quad , \quad d = C\bar{\mu}_5\bar{\mu}^2 + \bar{\mu}f(\bar{\mu}_5) + F(\bar{\mu}_5), \quad (19)$$

where f and F are arbitrary functions.

We are interested only in the ChME term which corresponds to the coefficient in front of B_μ . For this coefficient k_B in the lowest order in chemical potentials one gets:

$$\kappa_B = C\mu_5 + O(\mu_5^2, \mu\mu_5, \mu^2). \quad (20)$$

Higher-order terms in k_B are not fixed by this calculation. It's worth pointing out that the coefficients in front of ω^μ have uncertainty even in the lowest order while in [3] coefficient before ω^μ in the chiral current is uniquely fixed.

It is readily seen that there is a term proportional to the magnetic field in the vector current. This term corresponds to the ChME in the hydrodynamical model. Substituting the anomalous coefficient in front of $(E \cdot B)$ by $\frac{e^2}{2\pi^2}$ we finally get in the lowest order:

$$\Delta j^\mu = \frac{e^2\mu_5}{2\pi^2}B^\mu. \quad (21)$$

The answer for an arbitrary N_c is obtained by simple multiplication.

3 Conclusion

To summarize, it follows from the consideration of the entropy that the chiral magnetic effect is not vanishing in the hydrodynamical approximation. The corresponding coefficient c_{ch} , see (2), is expandable in the chemical potentials. The first, linear term is uniquely fixed and coincides with the result for non-interacting particles. Unlike the free-particle case, further terms in the expansion are not vanishing, generally speaking. They seem to be not fixed, however, by the general thermodynamic relations. The hydrodynamical approximation addresses physics in the infrared region of energies and is sensitive, generally speaking, to all orders of perturbation theory. Our result suggests, therefore, that there exists a new non-renormalization theorem for the coefficient c_{ch} in the linear in the chemical potential approximation. Besides the result itself which looks quite nonpresumable, one could wonder about power of pure thermodynamics in deriving of highly non-trivial relations in the field theory.

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